SYNCHRONIZED VORTEX SHEDDING AND SOUND GENERATION IN A CORRUGATED PIPE: A GLOBAL STABILITY APPROACH

Russo Serena\textsuperscript{1}, Giannetti Flavio\textsuperscript{1}, Luchini Paolo \textsuperscript{1} & Fabre David \textsuperscript{2}

\textsuperscript{1}Dipartimento di Ingegneria Industriale (DIIN), Università di Salerno, Italy
\textsuperscript{2}Institut de Mécanique des Fluides de Toulouse (IMFT), Université de Toulouse, France

INTRODUCTION

Corrugated pipes, such as gas hoses or electrical sheaths, are objects which combine a local rigidity and global flexibility, and are thus largely used in a number of engineering applications. At critical conditions, the flow through such a pipe can produce a loud whistling sound. This phenomenon is undesirable in industrial application because it can result in strong mechanical vibration and damage to the structure. On the other hand, the phenomenon is also at the origin of a musical toy called the “hummer” (sometimes also described as "the voice of the dragon") in which the flow is produced by centrifugal effect when swirling the pipe above one’s head.

The mechanism responsible for such a whistling has made the object of recent experimental and numerical studies \cite{4, 5, 6}. The origin of the mechanism lies in a coupling between vortex shedding occurring in a periodic way above the cavities which constitute the wall of the pipe, and an acoustic standing wave. The latter has the effect of stimulating the emission of vortices in a synchronized way, which in turn provide energy to the acoustic wave. The whole phenomenon can thus be described as a SASER (for “Sound Amplification by Stimulated Emission of Radiation” \cite{6}). Since the wavelengths corresponding to the acoustic part of the flow are much larger than the size of the corrugations, the flow at the scale of a single (or a few) cavities can be assumed, as a first approximation, to be incompressible. A recent experimental and numerical study \cite{3} has explored the interaction between adjacent cavities, and has shown that depending on the cavity spacing, the interaction can be constructive (when vortices are shed in a synchronized way) or destructive (when vortex shedding occurs with a phase opposition between adjacent cavities). The objective of the present work is to investigate such couplings using a global stability approach of the flow at the size of the cavities.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{Structure of the base flow for \(Re = 3000\). Color : iso-levels of vertical velocity; lines : streamlines.}
\end{figure}

APPROACH AND FIRST RESULTS

In a first step, we have chosen to work with a two-dimensional channel, between a straight lower wall and an upper one with a periodic shape, displaying large cavities. Because of the periodicity hypothesis, we can work in a reduced domain, but in order to capture both in-phase and out-of-phase shedding modes, we consider an elementary cell comprising two successive corrugations. The computations were done using two numerical methods, both developed and validated in the study of vortex shedding behind bluff bodies. The first one is a finite-difference code, as described in \cite{2}. The second one is a finite-element formulation, using the software FreeFem++, as described in \cite{1}. We apply a classical global stability approach to the problem by expanding the flow as follows:

\[
[u, p] = [u_0, p_0] + \epsilon [u_1, p_1] e^{(\sigma + i\omega)t}.
\]

Here \([u_0, p_0]\) is a steady solution of the incompressible Navier-Stokes equation in the elementary, periodic cell, which constitutes the "base flow" of the stability study. This base-flow, computed through Newton iteration, is depicted in
The perturbations are sought in eigenmode solution, with spatial structure $[u_1, p_1]$, amplification rate $\sigma$ and oscillation rate $\omega$. The computations are done using a shift-and-invert strategy. As an illustration, we depict in figure 2 the two most amplified modes. In this case, the most amplified mode is of the "in-phase" kind while the second one is of the "out-of-phase" kind (for the same condition as in figure 1).

We will explore the competition between the two kinds of unstable modes as the geometrical parameters (length and depth of the cavities, spacing between the cavities) are varied.

**Figure 2.** Structure of the two most unstable modes for $Re = 3000$, depicted through iso-levels of vertical velocity. (a) and (b): in-phase mode (two instant of the cycle separated by a quarter of period); (c) and (d): out-of-phase mode (idem).

### NEXT STEPS

Beyond the two-dimensional case displayed above, we will also consider the more representative case of a three-dimensional, axisymmetric pipe. In this case, in addition to the issue of synchronization between adjacent cavities, one may get the competition between axisymmetric modes (with azimuthal wavenumber $m = 0$), where the vortex shedding is synchronized all along the edge of the annular cavities, and non-axisymmetric modes (with azimuthal wavenumber $m = 1$), where the vortex shedding happens in an anti-phase way on opposing sides of the pipe. Implications on the sound production will be discussed. Finally, we are also working towards a more global approach to the problem, using the full compressible equations and considering a longer domain consisting of a large number of cavities.

### References


